The New Mexico alpha-omega Dynamo Experiment: Modeling Astrophysical Dynamos

S. A. Colgate[1,2], *V. I. Pariev*[2,3], *H. F. Beckley*[1], *R. Ferrel*[1], *V. D. Romero*[1], *J. C. Weatherall*[1]

[1] *Department of Physics, New Mexico Institute of Mining and Technology, NM 87801, USA*

[2] *Theoretical Astrophysics Group, T-6, MS B288, Los Alamos National Laboratory, Los Alamos, NM 87545, USAconstruct*

[3] *P. N. Lebedev Physical Institute, Leninsky Prospect 53, Moscow 117924, Russia*

A magnetic dynamo experiment is under construction at the New Mexico Institute of Mining and Technology. The experiment is designed to demonstrate in the laboratory the $\alpha\omega$ magnetic dynamo, which is believed to operate in many rotating and conducting astrophysical objects. The experiment uses the Couette flow of liquid sodium between two cylinders rotating with different angular velocities to model the ω–effect. The α–effect is created by the rising and expanding jets of liquid sodium driven through a pair of orifices in the end plates of the cylindrical vessel, presumably simulating plumes driven by buoyancy in astrophysical objects. The water analog of the dynamo device has been constructed and the flow necessary for the dynamo has been demonstrated. Results of the numerical simulations of the kinematic dynamo are presented. The toroidal field produced by the ω–effect is predicted to be $B_\phi \simeq (R_m/2\pi)B_{poloidal} \simeq 20 \times B_{poloidal}$ for the expected magnetic Reynolds number of $R_m \sim 120$. The critical rate of jets necessary for the dynamo self-excitation is predicted from the calculations to be a pair of plumes every 4 revolutions of the outer cylinder. For reasonable technical limitations on the strength of materials and the power of the drive, the self-excitation of the dynamo appears to be feasible.

Introduction Recent years have been marked by exciting developments in the field of MHD dynamos, in particular the experimental realization of homogeneous dynamos was achieved. After many years of research and preparations, an exponentially growing dynamo mode was observed in the experiment conducted in a liquid-sodium facility in Riga, Latvia [1, 2]. This experiment reproduces the simplest dynamo flow proposed by [3] and also subsequently by [4] in the laboratory. Another successful dynamo experiment of a different type was built in Karlsruhe, Germany. This experiment verifies the ability of a regular spatial arrangement of vortices to amplify the magnetic field. The growth of the magnetic field starting from the initial seed value of ≈ 1 Gauss up to ≈ 70 Gauss was observed in Karlsruhe experiment [5]. The magnetic field reached the back-reaction, saturated limit, and the excitation of the non-axisymmetric mode predicted by the theory was observed. There are a number of other dynamo experiments, which are under preparation or discussion. Each of these experiments is designed to test different flow patterns capable of dynamo action. All these experiments (except Karlsruhe experiment) are designed to use axisymmetric rotating flows, either stationary or non-stationary. We designed another kind of dynamo experiment, which will use essentially non-axisymmetric, non-stationary flows.

Here, because the flow is non-stationary and non-axisymmetric, then in both cases the field averaged over many plume ejections will approach a near steady state of axisymmetric symmetry. The flux from such a dynamo thus may simulate the astrophysical fluxes observed on large scales.

Real dynamos operate in Nature on astrophysical scales, in planets, convective envelopes of stars, galactic discs, accretion discs, and, possibly, on the largest scale in the clusters of galaxies. Parker [3], suggested the $\alpha\omega$ dynamo as an explanation of the large scale fields of many astrophysical dynamos. Here, cyclonic or anticyclonic plumes are the primary source of the helicity. In the case of planetary and stellar dynamos such plumes are believed to be rising and sinking convective cells [6, 7]. Here we invoke for both the experiment and the theory of the accretion disk dynamo, anticyclonic plumes because of their finite rotation angle, $\simeq \pi/2$ radians.

In the case of Galactic dynamos, supernovae explosions can also produce the necessary anticyclonic motions [8, 9, 10]. Finally, the most energetic dynamos in the present day universe should exist in the accretion discs around black holes in the nuclei of active galaxies [11]. The magnetic dynamo in the black hole accretion discs in the centers of galaxies could be due the stars passing through the accretion disc and producing the plumes of heated gas. These plumes rise above the disc plane, expand and produce anticyclonic motion necessary for the operation of an $\alpha\omega$ dynamo [12, 13]. The New Mexico dynamo experiment is designed to demonstrate the excitation of the astrophysical-type dynamo due to non-axisymmetric plumes rising through the differentially rotating liquid sodium. The design and initial construction phase of this experiment is underway at the New Mexico Institute of Mining and Technology located in Socorro (USA). Those involved in the conceptual design, engineering, mechanical design, and theoretical considerations of the experiment are associated with the New Mexico Institute of Mining and Technology and Los Alamos National Laboratory both located in the State of New Mexico, and so we refer to the experiment as the New Mexico Dynamo Experiment.

The experiment was designed after a water visualization experiment confirmed the existence of the two fundamental fluid motions necessary for a laboratory experiment to demonstrate the $\alpha\omega$ dynamo. These are: (1) the maximum differential shear and stability of Couette flow and (2) the production of axially aligned diverging plumes from pulsed jets and their subsequent anticyclonic and limited rotation relative to a rotating frame. Far more accurate measurements will be possible of both these flows in the completed dynamo apparatus, but we include here the qualitative confirmation of these fundamental flows as a basis of the design. In the Theory section we will show how these two orthogonal flows make an $\alpha\omega$ dynamo when produced in a conducting fluid of sufficiently large magnetic Reynolds number.

The plan of this paper is to briefly discuss how such an $\alpha\omega$ dynamo works in Sec. 2 and then in Sec. 3 to briefly review the practicality of creating the necessary flow geometry as demonstrated in water visualization experiments. In Sec. 4 we review the actual design and construction features of the experiment. In Sec. 5 we develop the all important numerical simulation of the dynamo growth rate with boundary conditions expected to be achieved in the experiment. We conclude in Sec. 6 with the status of the construction and conclusion.

1. How the $\alpha\omega$ dynamo works: Fig. 1(a) shows how the $\alpha\omega$ dynamo works in the New Mexico Dynamo Experiment and Fig. 1(b) shows how the $\alpha\omega$ dynamo works in the accretion disk forming the massive, central galactic black hole. In Fig. 1(a) differential rotation is established in the liquid sodium between two rotating cylinders as limiting stable Couette flow, $\Omega \propto 1/R^2$, by driving $\Omega_1 = 4\Omega_0$ where $R_0 = 2R_1$ and for the disk Fig. 1(b) as Keplerian rotation, $\Omega \propto 1/R^{-3/2}$, around the central mass or the black hole. This differential rotation wraps up the radial component of an initial poloidal field Fig. 1(b(A)) either made with coils or an infinitesimally small, $< 10^{-19}$ G seed field from density structure at decoupling. The resulting toroidal field becomes stronger than the initial poloidal field Fig. 1(b(B)) by $B_{toroidal}/B_{poloidal} = n_{\Omega} B_{poloidal}$, where in equilibrium with resistive decay, the limiting number of turns becomes $n_{\Omega} \simeq R_{m,\Omega}/2\pi$, and $R_{m,\Omega} = v_0(R_0 - R_1)/\eta$ is the magnetic Reynolds number. This multiplication factor depends upon the resistivity of liquid sodium or for the disk, upon the resistivity of the ionized and turbulent plasma. Then a driven pulsed jet or a collision with the disk by a star, Fig. 1(b(C))] causes a plume to rise either towards the end plate or above the disk with the corresponding displaced toroidal flux forming a loop of toroidal flux. The radial expansion of the plume material causes the plane of this loop to untwist or rotate differentially about its own axis relative to the rotating frame so that the initial toroidal orientation of the loop is transformed to a poloidal one, Fig. 1(b(D)).

Resistive diffusion in liquid sodium metal or reconnection in the ionized plasma of the disk allows this now poloidal loop to merge with the original poloidal field. For positive dynamo gain, the rate of addition of poloidal flux must be greater than its decay. It is only because the toroidal multiplication can be so large or that $R_{m,\Omega}$ can be so large that the helicity necessary for gain of the $\alpha\omega$ dynamo can be much smaller and episodic.

2. The Water Visualization Experiment: The water visualization experiment consists of two parts, the first establishing the Couette flow and showing that it is stable, and the second demonstrating the plume-like behavior produced by pulsed jets. Fig. 2(A) shows a photograph of the experiment where two cylinders of plexiglass^R of diameter 15 and 30 cm are rotated at various speeds of the order of 1 Hz and pulsed jets of water are injected from a plenum, in turn supplied by pulses of low pressure air. Digital video recording of the surface contour is shown from an angle and from the side in Fig. 2(B)&(C), which demonstrates the hyperbolic contour expected from limiting, maximum shear, stable Couette flow, $\Omega_1 = \Omega_0(R_0/R_1)^2$. The parabolic profile of the water in solid body rotation within the inner cylinder is just visible. Fig. 3(A) shows a schematic of the apparatus for observing the plumes with a co-rotating camera. When plumes are injected in stationary flow with an imbedded linear array or "line" of hydrogen bubbles, Fig. 3(B), one observes from the side the outline of a rising, diverging vortex, which in turn simulates a diverging plume. When the same plume is observed from the top or axial end and when both the camera and plume are rotating at Ω_0, then one observes the differential, anticyclonic rotation of the same imbedded line of bubbles, Fig. 3(C). In addition, one observes that the differential rotation of Couette flow both speeds up the anticyclonic rotation and the dispersal of the plume. With these observations, the design of the experiment was undertaken.

3. Design of the experiment The experiment consists of two coaxial cylinders rotating with different angular velocities, Ω_0 at R_0, the outer radius and Ω_1 at R_1, the inner radius, where $R_0/R_1 = 2$. The space between cylinders will be filled with liquid sodium with a small "topping" of mineral oil. The volume of sodium between the cylinders is limited by two end plates. One of the plates is solid while the other plate (referred to as the port plate) has two circular openings symmetric with respect to the rotation axis of the apparatus and of diameter $R_{port} = (1/3)R_0$. These ports are in turn connected to a plenum, supplied with pulsed pressurized liquid sodium, that forms the pulsed jets. There is also an annular space between the outer cylinder and the plenum. These periodic pulses of sodium flow are driven through the ports by a piston inside the plenum. Sodium flows out of the circular ports and returns back to the plenum volume through the annular space between the plenum cylinder and the outer cylinder. During injection of the jets, the resulting plume expands. Due to the Coriolis force acting on the expanding plume in the rotating frame, the plume rotates in the direction opposite to the direction of the rotation of the vessel in the same fashion as shown in Fig. 3(C). Such a rotating motion of the plume in the rotating frame and its axial translation corresponds to an unwinding helical motion. This helical motion produces the poloidal magnetic flux out of toroidal magnetic flux or α-effect. The design drawings of the experiment are shown in Fig. 4 & Fig. 5.

In both figures the magneto active volume with fluid sodium is the annular space between the differentially rotating inner and outer cylinders and between the left hand end plate and the middle port plate. Figure 4 is the first stage, the Ω-Phase of the experiment and presently under construction. The plume generating mechanism or α-Phase will be added in the second stage, Fig. 5. The purpose of first stage, without the plumes, is to demonstrate the production of the toroidal field from the velocity shear and the applied poloidal field. In addition some experiments will be performed to investigate the magneto rotation instability (MRI) using an axial applied field rather that a poloidal field with a radial component.

The second stage of the experiment will add the drive mechanism to produce the plumes and possibly lead to positive dynamo gain. Here the solid mid-plate has two ports for producing the pulsed jets from pressure generated in the plenum by a driven piston on the right. The port plate, plenum and drive mechanism are rigidly connected and rotate at Ω_0. The inner cylinder will be rotating faster, Ω_1, than the outer cylinder in order to create the differential rotation. It is driven by its own high speed shaft. The secondary drive shaft, gears, belts and motor are not shown. Initially a 50 kW motor, pulsed to 100 kW will be used. Greater power can be applied if needed, but the basic limitation is the mechanical strength of the outer cylinder which contains the centrifugal pressure of the sodium. We have used aluminum for the construction contrary to the usual practice of stainless steel in reactor coolant technology. Here the temperatures are very much less and water vapor is excluded by mineral oil. In addition the useful experimental life of the apparatus is short such that the aluminum corrosion by NaOH will be negligible.

Since the end walls rotate at a different rate from the sodium in Couette flow, the velocity shear at the walls would produce eddy currents between the sodium and the conducting aluminum walls whenever an axial field component penetrates both. Thus in this case, no conduction is desired between the end walls and sodium and so this interface will be insulated. This condition particularly applies when the field is made purely axial as for the measurements of the magneto rotational instability, or MRI measurements. On the other hand, electrical conduction between the sodium and the inner and outer conducting aluminum cylinders is desired in order to maximize the magnetic shear for the production of the enhanced toroidal field. Finally we recognize that the Ekman layer flow at the end walls, a fast radial flow in a thickness $\delta_{Eckman} \simeq R_0 Rey^{-1/2}$, is so thin, $\delta_{Eckman} \simeq 3 \times 10^{-4} R_0$, that its electrical conduction current is negligable compared to the primary currents induced by the Couette flow.

The sodium will be heated and liquefied by the hot mineral oil driven by a recirculating system through the space inside the inner cylinder. This oil flow also serves to maintain the thermal balance of the sodium, slightly above the melting temperature and also to prevent the further heating of the sodium due to the friction heat produced primarily in the Ekman layers. In addition, the non-recirculating mineral oil, used to "top" the sodium metal in the apparatus, isolates the liquid sodium from the rotating seals and the one internal bearing. This isolation of liquid sodium from the seals and bearing takes place because the density of oil is 0.86 of the density of the sodium at 110° C, and so the oil will float to the central axis of the rotating device. The resulting oil coating will also ensure the isolation of liquid sodium from the air in the case of a minor spill as previous experience has shown, [15].

The maximum shear is desired in the rotational flow between the cylinders, yet maintaining stable flow. Therefore, in order to maximize the shear and maintain minimum fluid drag, or minimize the torque with the walls and hence, power, one should use Couette flow at the margin of stability, i.e. when $\Omega_1 R_1^2 = \Omega_0 R_0^2$. The New Mexico Dynamo Experiment is designed to have this marginally stable ratio of angular velocities of the cylinders. Namely, $R_0/R_1 = 2$ and $\Omega_1/\Omega_0 = 4$. In the case of marginally stable Couette flow the angular velocity profile becomes

$$\Omega = \frac{\Omega_1 R_1^2}{r^2} = \frac{\Omega_0 R_0^2}{r^2}. \tag{1}$$

The geometrical parameters of the experiment are: inside radius of the outer cylinder is $R_0 = 30.5\,\mathrm{cm}$, wall thickness of the outer cylinder is $\Delta R = 3.2\,\mathrm{cm}$, length of the test-volume is $L = 30.5\,\mathrm{cm}$, wall thickness of the port and end plates is $\Delta L = 3.2\,\mathrm{cm}$, radius of the inner cylinder is $R_1 = 15.25\,\mathrm{cm}$, the length of the space filled with liquid sodium in the plenum behind the port plate is $L_1 = 35.6\,\mathrm{cm}$, radius of the plume ports is $r_p = 4.9\,\mathrm{cm}$, the radial distance from the center of the plume port to the rotation axis of the cylinder is $r_0 = R_1 + r_p = 20.15\,\mathrm{cm}$, the width of the annular space between the outer cylinder and the plenum is $s = 2.6\,\mathrm{cm}$. One desires a size as large as possible, but is limited by the costs implied by available standard bearings, drive belts, material handling and machine tools.

The material for both cylinders, port and end plates, and the two ported reservoir plenum cylinders is aluminum alloy 5083-H3. This alloy has the necessary strength to sustain the centrifugal pressure of rotating sodium at the required temperatures. It is widely used in industry and properties well known. The high- and low-speed drive shafts, left and right flanges are made of steel. The kinematic viscosity coefficient of liquid sodium at 110° C is $\nu = 7.1 \cdot 10^{-3}\,\mathrm{cm^2\,s^{-1}}$. The magnetic diffusivity of liquid sodium is $\eta = 810\,\mathrm{cm^2\,s^{-1}}$, the magnetic diffusivity of the aluminum alloy walls is $\eta_{Al} = 650\,\mathrm{cm^2\,s^{-1}}$.

The analysis of stresses and energy dissipation in the experiment indicates that the maximum frequency of rotation of the outer cylinder is limited by the yield strength due to the centrifugal pressure and is $\leq 33\,\mathrm{Hz}$. This corresponds to $\Omega_0 \leq 207\,\mathrm{s^{-1}}$. The angular velocity of the inner cylinder is always four times larger than the rest of the device and is limited by $\Omega_1 \leq 828\,\mathrm{s^{-1}}$. At this angular velocity the average wall stress in the outer wall, ΔR at R_0 is $\sim 1/4$ of the yield strength.

We define the global hydrodynamic Reynolds number of the Couette flow in the experiment as

$$\mathrm{Re}_\Omega = \frac{\Omega_0 R_0 (R_0 - R_1)}{\nu}. \tag{2}$$

The magnetic Reynolds number for the rotational Couette flow is defined in a similar way as

$$\mathrm{Rm}_\Omega = \frac{\Omega_0 R_0 (R_0 - R_1)}{\eta}. \tag{3}$$

Then, the ratio $\mathrm{Re}_\Omega/\mathrm{Rm}_\Omega = \nu/\eta = \mathrm{Pm}$ is a magnetic Prandtl number. For sodium at 110° C one has $\mathrm{Pm} = 8.8 \cdot 10^{-6}$. The maximum Reynolds numbers corresponding to the maximum possible frequency of rotation are $\mathrm{Re}_\Omega \approx 1.3 \cdot 10^7$ and $\mathrm{Rm}_\Omega \approx 120$. This magnetic Reynolds number is higher than the one achieved in the successful dynamo experiments carried out so far, and we hope that this will allow us to investigate a wider spectrum of the behavior of conducting liquids. Particularly, the observations of MHD turbulence may be possible. At the same time, slower rotation rates can produce flows with as low magnetic Reynolds number as desired.

The experiment will also have current coils to produce an external magnetic field within the magneto active volume. These fields will be both with primarily radial (poloidal) or primarily axial components depending upon the emphasis of the the experiment, i.e., toroidal gain or MRI. All three components of the magnetic field will be measured by miniature Hall-effect detectors placed at a various radii inside an aerodynamically shaped probe inside the sodium. In the second stage, with driven plumes, we hope to measure the flux and its orientation transported by the plumes or α effect. These measurements will be performed by an array of magnetic detectors placed on the surface of the end plate opposite a plume. The fast response of the Hall-effect detectors, micro seconds, will allow one to record the detailed time evolution of the magnetic field produced by a single plume. A radial array of 5 pressure transducers will allow an accurate measurement of the pressure profile and thus a measurement of the Couette profile.

4. Numerical simulations of the dynamo At the current stage of numerical modeling of the New Mexico Dynamo Experiment we assumed the following simplified model. We consider the flow of the liquid sodium only inside the cylindrical, annular, space bounded by the radii R_1 and R_0 and the end plate and port plate at $z = L/2$ and $z = -L/2$. The walls are assumed perfectly conducting. In view of the conductivity of aluminum walls being higher than the sodium, this assumption qualitatively predicts the evolution of magnetic fields and the growth rate of the dynamo. However, to obtain more accurate results, one needs to consider more realistic boundary conditions taking into account the finite thickness of the walls, the insulating air, i.e., vacuum, outside the device, and the plenum filled with liquid sodium at one end. We also assume a simplified kinematic model for the flow and do not actually solve the hydrodynamic equations. In other words we do not take into account the pondermotive force ($\mathbf{j} \times \mathbf{B}$).

The kinematic dynamo equation is

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times (\eta \nabla \times \mathbf{B}) + \nabla \times (\mathbf{v} \times \mathbf{B}), \tag{4}$$

where η is the a coefficient of magnetic diffusivity, $\eta = \dfrac{c^2}{4\pi\sigma}$. Instead of solving the dynamo equation for $\mathbf{B}$ we introduce potentials $\mathbf{A}$ and φ. Using the gauge condition $c\varphi - \mathbf{v} \cdot \mathbf{A} + \eta \nabla \cdot \mathbf{A} = 0$ one can derive the following equation for the evolution of the vector potential $\mathbf{A}$

$$\frac{\partial A^i}{\partial t} = -A_k \frac{\partial v^k}{\partial x^i} - v^k \frac{\partial A^i}{\partial x^k} + \eta \frac{\partial^2 A^i}{\partial x^k \partial x_k}, \tag{5}$$

where the magnetic diffusivity η is assumed to be constant throughout the cylinder and the coordinate notations refer to a Cartesian coordinate system x^i. We use our 3D kinematic dynamo code to solve Eq. (5). Then, the magnetic field can be obtained at any time by taking the curl of $\mathbf{A}$. The code is written in cylindrical coordinates and uses an explicit scheme with the central spatial differencing in the advection term and the standard nine points stencil for the diffusion term. We assume all boundaries of the cylinder ($r = R$, $z = -L/2$, and $z = L/2$) to be perfect conductors. Then, the boundary conditions for **A** at perfectly conducting boundaries compatible with the gauge can be chosen as follows: the components of **A** parallel to the boundary are zeros and the divergence of **A** at the boundaries is zero. This gives three boundary conditions for three components of the vector potential.

We use the following dimensionless units: the unit of length is the outer radius of the test volume, R_0, the unit of velocity is the azimuthal velocity of the outer cylinder, $\Omega_0 R_0$. Therefore, the outer cylinder makes one revolution during the dimensionless time 2π, the inner cylinder makes one revolution during the dimensionless time $\pi/2$. The critical Couette velocity profile is assumed, $\Omega = 1/r^2$ in our units. The inner radius is 1/2 in our units, the length of the test volume is 1. In order to account for the conducting material inside the inner cylinder we extended our computational region toward $r = 0.5$ and assumed solid body rotation for $r < 0.2$ with the angular velocity equal to the angular velocity of the inner cylindrical wall of the test volume. We work in cylindrical coordinates with the axis of rotation centered on the axis of symmetry of the device. The $z = -0.5$ plane is the port plate of the test volume, the $z = 0.5$ plane is the end plate of the test volume.

We assume that there is a bias magnetic field produced by external coils and frozen in the ideally conducting boundaries. To approximate this field we choose the initial conditions in the form $A_r = 0$, $A_z = 0$, $A_\phi = r(z + 0.5)$. Then, the initial magnetic field is $B_r = -r$, $B_z = 2(z + 0.5)$, $B_\phi = 0$. This poloidal field satisfies the equation $\nabla^2 \mathbf{B} = 0$ and, therefore, is unchanged if only rotation is present.

First, we do simulations without plumes, when the flow velocity inside the device is Couette flow given by Eq. (1). Both the poloidal and stationary state toroidal fields are shown in Fig. 6 for $\mathrm{Rm}_\Omega = 120$. The ratio of the toroidal field to the poloidal field is ~ 20 and depends on the position of measurement inside the test volume. The toroidal field near the end plate has the opposite sign from the toroidal field in the middle of the test volume. The reversal of the sign of the toroidal magnetic field can be understood in terms of the conservation of total flux of the toroidal magnetic field through the cross section of the cylindrical computational space. Since this space is bounded by an ideal conductor, the total magnetic flux cannot change. Initially, the toroidal magnetic field was zero, so the total net flux of the toroidal magnetic field should remain zero. Therefore, regions of the magnetic field with different signs must exist in the rotating liquid. Of course, there is no actual discontinuity of the fluid velocity near the end plates of the test volume. Instead, the Eckman boundary layers develop at the end plates. However, the approximation of the Eckman layer by a mathematical discontinuity of the toroidal velocity near the end plates has very little effect on the structure of the magnetic fields excited in the conducting rotating fluid under the conditions of the experiment.

Next, we modeled the kinematic dynamo produced by jets of sodium together with the Couette differential rotation. The plume flow is interposed onto a background Couette differential rotation occupying the whole computational domain $\mathbf{v}_c = \dfrac{1}{r}\mathbf{e}_\phi$. A jet of liquid sodium is simulated by a vertically progressing cylinder of radius r_p in the co-rotation frame that starts at the port plate of the disk located at $z = -L/2$ and emerges to a height of $z = L/2$. At the same time the cylinder axis moves with the local Couette velocity. By the time the plume reaches its highest point the jet cylinder rotates by π radians. The cylinder does not rotate with respect to the rest frame. Therefore, in the frame rotating with the Keplerian velocity, it untwists by π radians during the time it rises. At the same time the axis of the cylinder moves by π radians around the central axis participating in the rotation with the Couette speed. The length of the cylinder increases with time and its velocity, $v_{pz} \approx v_c$. The vertical velocity of the liquid inside the cylinder is constant and is equal to v_{pz}. After the time the plume rotates by π it is stopped and the velocity field is restored to be pure Couette differential rotation everywhere. This very simplified flow field captures the basic features of the actual and complicated flow produced by the driven sodium jets. We note, however, that in view of the water visualization experiments, Fig. 2 and Fig. 3, that this approximation of constant radius plumes may be a conservative approximation, because the actural plumes diverge in radius leading to a progressive increase in $R_{m,plume}$.

Since Eq. (5) requires spatial derivatives of the velocity components, we apply smoothing of discontinuities in the flow field described above. Also we introduce smooth switching on and off of the jets in time. For all three components of velocity v^k we use the same interpolation rule, which for two plumes is

$$v^k = v^k_{in1} s_1 + v^k_{in2} s_2 + (1 - s_1 - s_2) v^k_{out}. \tag{6}$$

Here $s_1(r, \phi, z, t)$ and $s_2(r, \phi, z, t)$ are smoothing functions for plume 1 and 2 correspondingly. Each function s is close to 1 in the region of space and time occupied by the plume and is close to 0 in the rest of space and during times when the plume is off. Transition from 1 to 0 happen in a narrow layer at the boundary of the plume and during the interval of time short compared to the characteristic time of the plume rise. v^k_{in1} and v^k_{in2} are velocities of the flow of plumes 1 and 2, v^k_{out} is the velocity of the flow outside the regions occupied by the plumes. For spatial derivatives of the velocity components, one has from expression (6)

$$\begin{aligned} \frac{\partial v^k}{\partial x^i} &= \frac{\partial s_1}{\partial x^i}\left(v^k_{in1} - v^k_{out}\right) + \frac{\partial s_2}{\partial x^i}\left(v^k_{in2} - v^k_{out}\right) + \\ & s_1 \frac{\partial v^k_{in1}}{\partial x^i} + s_2 \frac{\partial v^k_{in2}}{\partial x^i} + (1 - s_1 - s_2) \frac{\partial v^k_{out}}{\partial x^i}. \end{aligned} \tag{7}$$

It is easy to extend this approach for the case of arbitrary number of plumes.

Let us assume that the cylindrical jet, going in the positive direction of the z axis, is launched at the position of the axis of the jet at $r = r_0$ and $\phi = \phi_0$. Let us denote this plume as number 1 and the symmetric plume also ejected at the same time as number 2. Then, after time $(t - t_p)$ from the starting moment of the plume $t = t_p$, its position is

$$\phi_1 = \phi_0 + (t - t_p) r_0 \Omega_{c0}, \tag{8}$$

where $\Omega_{c0} = \Omega_c(r_0)$ is the Couette angular rotational velocity at $r = r_0$. The position of the axis of the symmetric plume is

$$\phi_2 = \phi_1 + \pi. \tag{9}$$

The radii of both plumes are r_p. The originating surface of the plumes 1 and 2 is at the port plate at $z = -L/2$. The leading surface of the plume 1 is at $z_1 = -L/2 + v_{pz}(t - t_p)$ and the leading surface of the plume 2 is $z_2 = z_1$. The velocity field inside the upward jet number 1 is

$$v^r_1 = r_0 \Omega_{c0} \sin(\phi - \phi_1), \tag{10}$$

$$v^\phi_1 = r_0 \Omega_{c0} \cos(\phi - \phi_1), \tag{11}$$

$$v^z_1 = v_{pz}. \tag{12}$$

The same velocity field given by expressions (10–12) is inside the jet number 2 with the obvious replacement of ϕ_1 by ϕ_2. We choose the following interpolation functions

$$s_1 = \left(\frac{1}{2} + \frac{1}{\pi}\arctan\frac{r_p^2 - {r'_1}^2}{2r_p\Delta}\right)\left(\frac{1}{2} + \frac{1}{\pi}\arctan\frac{(z+L/2)(z_1 - z)}{\Delta\sqrt{(z_1+L/2)^2+\Delta^2}}\right)S(t) \quad (13)$$

and

$$s_2 = \left(\frac{1}{2} + \frac{1}{\pi}\arctan\frac{r_p^2 - {r'_2}^2}{2r_p\Delta}\right)\left(\frac{1}{2} + \frac{1}{\pi}\arctan\frac{(z+L/2)(z_2 - z)}{\Delta\sqrt{(z_2+L/2)^2+\Delta^2}}\right)S(t). \quad (14)$$

Here ${r'_1}^2 = r_0^2 + r^2 - 2r_0 r\cos(\phi - \phi_1)$ is the distance from the axis of the plume 1, ${r'_2}^2 = r_0^2 + r^2 - 2r_0 r\cos(\phi - \phi_2)$ is the distance from the axis of the plume 2, Δ is the thickness of the transition layer of the functions s_1 and s_2 from their value 1 inside the plume to 0 outside the plume, $\Delta \ll r_p$. The square root expressions in z–parts of s_1 and s_2 ensure that the thickness of the transition layer in the z direction is never less than Δ, even just after the plumes are started, when the differences $(z_1 + L/2)$ and $(z_2 + L/2)$ are zero. We choose $\Delta = 0.01$.

The function $S(t)$ ensures a smooth "turning on" and "turning off" of the plumes at prescribed moments of time. If the plumes are to be started at $t = t_p$ and to be terminated at $t = t_d$ ($t_d > t_p$), then we adopt the following form of the function $S(t)$

$$\begin{cases} S(t) = 0, & \text{for} \quad t < t_p - \delta t/2, \\ S(t) = \frac{1}{2} + \frac{1}{2}\sin\left(\pi\frac{t-t_p}{\delta t}\right), & \text{for} \quad t_p - \delta t/2 < t < t_p + \delta t/2, \\ S(t) = 1, & \text{for} \quad t_p + \delta t/2 < t < t_d - \delta t/2, \\ S(t) = \frac{1}{2} - \frac{1}{2}\sin\left(\pi\frac{t-t_d}{\delta t}\right), & \text{for} \quad t_d - \delta t/2 < t < t_d + \delta t/2, \\ S(t) = 0, & \text{for} \quad t > t_d + \delta t/2. \end{cases}$$

where δt is the length of the transition period. $S = 0$ corresponds to the flow without plumes, $S = 1$ corresponds to the flow with plumes. One needs to ensure that $\delta t < t_d - t_p$. We took $\delta t = (t_d - t_p)/5$. The cycles with the cylindrical jets present are interchanged periodically with the cycles without such jets and with the pure Couette rotation only. The time between two subsequent ejections is Δt_p and we always have $\Delta t_p > t_d - t_p$, such that at any time only one pair of plumes are present.

We take the radius of the jet $r_p = 0.21$, the position of the center of the jet at $r_0 = 0.71$, and the vertical velocity of the flow inside the rising cylinder $v_{pz} = 0.63$. This geometry corresponds to the experimental setup described in Sec. 3. The radius of the jet is chosen to be the maximum possible, and the vertical velocity of the liquid in the jet requires only moderate power of the piston driving mechanism. The vertical velocity of the plume is chosen such that the plume reaches the end plate during the time when the fluid at the radius of the plume rotates by the angle π. During this same time the plume rotates clockwise by $\approx \pi$ radians in the local Couette rotating frame. Such a timing should maximize the release of the poloidal magnetic flux due to the diffusion out of the twisting plume into surrounding conducting fluid. After the plume reaches the end plate, the velocity field of the flow is smoothly set back to the pure Couette profile without further poloidal motions. There are two identical plumes ejected simultaneously through the two orifices located symmetrically with respect to the rotation axis of the device. We performed simulations with different time intervals between subsequent plume ejections. We looked at the various rates of: (1) one pair of plumes per one revolution of the outer cylinder (i.e., per 2π units of time), (2) one pair of plumes per two revolutions of the outer cylinder, (3) one pair of plumes per three revolutions of the outer cylinder, and so on, until the dynamo could no longer be excited. The typical curve of the energy growth of the magnetic field is presented in Fig. 7. The dependence of the growth rate of the dynamo vs. the rate of plumes (the rate is the inverse of the number of revolutions of the outer cylinder, N, per one ejected pair of plumes) is shown in Fig. 8. One can see that the threshold for the dynamo excitation is somewhere between one pair of plumes per 4 revolutions and one pair of plumes per 5 revolutions. The smallest point on the graph for $N = 5$ may not have converged numerically, and so is likely to be below the axis where $\gamma = 0$.

5. Present Status The first phase of the experiment, the Ω-Phase, is largely completed, Fig. 9. The electronics are still a major consideration where the data from roughly 120 detectors, magnetic, pressure and temperature, must be transmitted digitally to a computer from the rotating equipment.

6. Conclusions The numerical simulations along with the engineering design feasibility give us confidence that, with a continuing effort a positive gain $\alpha\omega$ dynamo can be made in the laboratory.

Acknowledgements This work was partially supported by the US Department of Energy, under contract W-7405-ENG-36 and NSF Grant 9900661. We particularly acknowledge the initial support of the New Mexico Institute of Mining and Technology, (NMIMT) through the Energetic Materials Research and Testing Center (EMRTC) that allowed this project to begin.

We would like to express our gratitude to J. M. Finn, LANL for numerous discussions on magnetic dynamos and help with the numerical code and Hui Li, LANL, for discussions on how this work joins to astrophysics.

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Received December 26, 2001

Figure 1: A schematic of the sodium dynamo experiment Fig. 1(a) in comparison to the accretion disk dynamo of Fig. 1(b). In both cases differential rotation in a conducting fluid wraps up an initial radial field component into a much stronger toroidal field. Then either liquid sodium plumes driven by pulsed jets or star-disk collisions eject and rotate loops of toroidal flux into the poloidal plane. Resistivity or reconnection merges this new or additional poloidal flux with the original poloidal flux leading to dynamo gain.

Figure 2: (A) The plume rotation experiment apparatus with its camera mounts, electrical communications, air supply, and the cylinder drive systems. (B) The surface of the maximum shear stable Couette flow. Note the smooth hyperbolic surface indicative of low turbulence. (C) The side view of the contour of the maximum shear, stable Couette flow. Note the hyperbolic surface external to the inner cylinder and the parabolic surface of the water inside the inner cylinder in solid body rotation.

Figure 3: (A) Schematic of the plume rotation experiment apparatus for viewing the rising plumes from the side and the plume rotation from above. (B) The entrained small bubbles of hydrogen from pulsed electrolysis outlines the rising plume without rotation from the pulsed jet. (C) The same plumes viewed from above in a rotating frame. The line of bubbles trace a progressive rotation of the plume as a function of rotation of the frame.

Figure 4: The detailed design drawing of the rotating components of the Ω-Phase of the New Mexico Dynamo Experiment. By comparing Figs. 4 and 5 the labeling of the various components can be compared and identified in the two drawings. The main cylinder of radius $R_1 = 30.5\,\mathrm{cm}$ rotates between two bearing mounts. In the Ω-Phase there is no hydraulic drive to produce the plumes even though the constructed aluminum parts of Fig. 5 show the port plate and two ported reservoir plenum cylinders. These parts are necessary to support the port plate which defines one end of the Couette flow annular experimental volume.

Figure 5: The design drawing of both the rotating components as well as the plume drive mechanism of both the α- and Ω-Phases of the New Mexico Dynamo Experiment.

Figure 6: The top panel shows the poloidal magnetic field and the bottom panel shows the contours of equal values of the toroidal magnetic field. The toroidal magnetic field is shown after the steady state between stretching and diffusion of the magnetic field has been reached.

Figure 7: Part of one simulation for one pair of plumes per three revolutions showing exponential growth of the dynamo. The small spikes on the graph are due to the action of each single plume. The slowly rising and decaying arches are due to the production of the toroidal field from the fraction of the poloidal field added by the plume.

Figure 8: The dependence of the growth rate of the dynamo on the plume rate. N is the number of revolutions of the outer cylinder from one ejection of a pair of plumes to the next ejection of a pair of plumes.

Figure 9: The plume port plate and reservoir plenum cylinders are shown yet to be mounted internally. The plume drive piston is not constructed in this phase. The plume port plate is supported by the reservoir plenum cylinder for greater rigidity. The rotating drive components of the experiment are mounted on the bearing supports. The massive bearing supports and base plate are designed to reduce the rotating vibration amplitude. The oscillating hydraulic drive mechanism for the jet–plume production is not constructed, awaiting the α-Phase of the experiment.